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ON THE MASS TRANSFER IN A SPRINKLED BIOFILTER

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The mass transfer from a water film to fixed fill grains (Rashig ring) has been investigated. It has been revealed that the film regime of flow is characterized by incomplete flow over the surface of Raschig rings. The mass transfer coefficient and the portion of the washed surface depending on the flow rate of water have been measured. The process of mass transfer in a freely falling film has been simulated, and a comparison between the theoretical and measured values of the mass transfer coefficient has been made.

Keywords: *biofilter, film flow, mass transfer coefficient.*

Introduction. Sewage treatment by industrial methods is among problems of vital importance [1–3]. Biological methods of water purification are among the most effective ones. One kind of water treatment bioreactors is column apparatuses with a fixed granular bed covered with a biofilm. Such biofilters are characterized by a large biofilm–water interface and a low energy input, since water flows through the apparatus without pressure, by gravity.

Water with a high content of impurities that are easy to oxidize biochemically is passed through a layer of particles with a film of microorganisms on the surface. In so doing, impurities dissolved in the water are transported into the biofilm, where they serve as a substrate for microorganisms (aerobic as a rule) processing it to the end product [1, 2]. The intensity of substrate processing by microorganisms is largely determined by the velocity with which it is carried from the water into the biofilm [1, 2, 4, 5], which in turn depends on the hydrodynamic conditions of the flow in the two-phase system [6]. For granular bed particles, one can use, for example, Raschig rings with a high specific surface.

Depending on the flow rate of the liquid and the properties of the biofilm surface (wettability, roughness) and structure, the following flow conditions may be realized: streams (rivulets), film flows (the running-off liquid covers the whole of the biofilm or a part of it), or complete flooding of the granular bed. In the first two kinds of flow conditions a fairly large space of pores containing air needed for aerobic biochemical reactions to proceed is left, and to reproduce this air, it is necessary to organize cocurrent or counter air flows.

In the literature [7–14], many formulas for the mass transfer coefficient β_w of a liquid flow with fill grains are known. In the general form, they can be given as

$$\mathrm{Sh} = \theta\,\mathrm{Sc}^k\,\mathrm{Re}_r^n\,, \tag{1}$$

where $\mathrm{Sh} = \beta_w\tilde{R}/D$; $\mathrm{Sc} = \nu/D$; $\mathrm{Re}_r = \dfrac{Q\tilde{R}}{\pi\tilde{R}_r^2\nu(1-\varepsilon)}$ denotes the Sherwood, Schmidt, and Reynolds criteria numbers. In the table for estimating β_w, values of the parameters k, n, θ from different literature sources are given.

First of all we note that the given formulas were obtained in investigations realized at complete filling of the bed pores. In a number of works [8, 10], the influence of the substrate (biofilm in the given case) on the mass transfer coefficient expressed in terms of the relation $(\tilde{R}/l)$ was studied.

Another distinguishing feature is the considerable spread in the exponents of all the given formulas. At filtration in a completely flooded granular bed there occurs a mass transfer between the walls of complex-geometry channels and the filtration flow of the liquid. Of no small importance here are the packing geometry and the stagnation

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TABLE 1. Parameters k, n, θ in (1) Used by Different Authors (for $\mathrm{Re_r} < 100$)

k	n	θ	Literature source	Note
1/3	4.33	$\approx 10^{-11}/\tilde{R}$	[7]	Flow through a water-saturated layer of granules in the channel
0.27	1/3	$\approx 1.61(\tilde{R}\ /l)^{1/3}$	[8]	Flow through the membrane
1/3	0.6	≈ 0.51	[9]	Fixed bed of the biofilter
1/2	1/2	$2.8(\tilde{R}\ /l)^{1/2}$	[10]	Bioreactor
1/3	1/3	Function of the biofilm structure	[11]	Fixed bed of the biofilter
1/3	1/4	≈ 5.7	[2]	Flow through a water-saturated layer of particles (the dependence was obtained in Jenings' theses (1975))
0.6	1/3	≈ 1.1	[12]	Flow through a water-filled layer of particles
1/3	1/3	≈ 0.55	[13]	Flow through a water-filled layer of particles
1/3	0.28	$\dfrac{\varepsilon}{(1-\varepsilon)^{0.28}}\left(\dfrac{R_{\mathrm{r}}}{\tilde{R}}\right)^{0.28}$	[14]	Flow through a water-filled layer of particles, $\mathrm{Re_r} < 20$

regions formed at contact points and interfaces where the mass transfer obeys laws other than the laws of developed convection in channels. This requires caution in choosing the experimental mass transfer measuring technique, in estimating the effective surfaces and their contribution to the mass transfer, and in interpreting the obtained results.

For calculations of the mass transfer in biofilters, of paramount importance is the dependence of the mass transfer coefficient on the feed rate of the substance, i.e., on the $\mathrm{Re_r}$ number. In fact, only this parameters can be varied effectively to attain optimum operation of the biofilter.

At a fairly low flow rate of water, which is dictated by the necessity to provide air inflow and the requirement of the largest possible contact surface with water, film flow past particles in the bed is possible (we observed this visually). In [15], we measured the intensity of mass transfer from the water film to the fill grains and obtained the dependence $\beta_{\mathrm{w}} \propto Q^{0.123}$, i.e., the dependence of Sh on $\mathrm{Re_r}$ is weaker that than given in the table, which is natural to associate with the film character of the flow past grains.

Finally, note the works in which the mass transfer from the liquid film to the solid surface of a vertical [16] and an inclined planes [17] was investigated theoretically and experimentally. The results point to the power law $\mathrm{Sh}(\mathrm{Re_r})$ with the exponent $n = 1/9$ in formula (1).

Below we consider experimentally and theoretically the problem of conveying the substance dissolved in the water film to the surface of Raschig rings past which it flows and which models the biofilm surface in a sprinkled biofilter. Particular consideration is given to the contribution of the inner surface of the run-through element of the film to the mass transfer. The obtained expression for the mass transfer is compared to the measurements made by us.

Theoretical Analysis. Let us assume on the basis of observations that the water flows along grains in the film regime by gravity, and the gaseous medium is stationary. The mass transfer process and the flow in the film will be considered to be stationary and the film hydrodynamically stabilized. And although, as was visually observed, at low flow rates of the liquid not the whole of the ring surface is washed and when the liquid flows from one ring to another the film breaks, the estimates given below show that recovery of the stabilized film characteristics occurs at a distance much smaller than the ring size.

In stabilized running down of the film on an inclined surface in a stationary gaseous medium a parabolic velocity profile of the liquid is realized [6]:

$$u = -\frac{\tilde{g}}{2\nu} y^2 + \frac{\tilde{g}\delta}{\nu} y \,. \tag{2}$$

Here $\tilde{g}$ takes into account the angle α between the grain surface and the averaged direction of the flow (Fig. 1) $\tilde{g} = \frac{2g}{\pi} \int_{\pi/2}^{0} \cos \alpha \, d\alpha = \frac{2}{\pi} g$. Hence the flow rate of the liquid per unit width of the film is

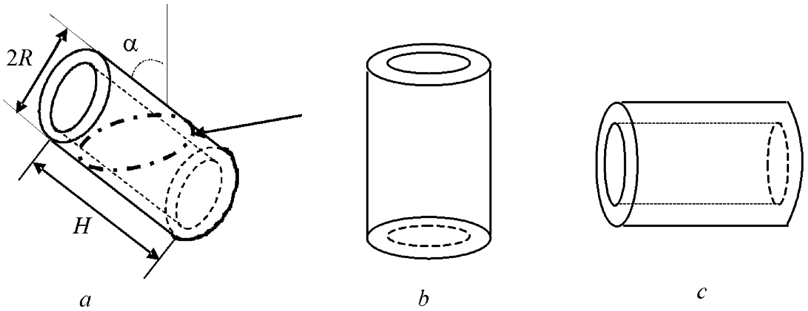


Fig. 1. Determination of the average perimeter of the Raschig ring in the fill: a) some random position characterized by angle α; b and c) extreme positions of the ring.

$$Q_2 = \int_0^\delta u dy = \frac{\tilde{g}}{3\nu}\delta^3 \tag{3}$$

or for the film thickness we have

$$\delta = \left(\frac{3Q_2\nu}{\tilde{g}}\right)^{1/3}. \tag{4}$$

The volume of the liquid running off the Raschig ring is equal to $Q_3 = 4\pi\tilde{R}Q_2$, where the effective radius $\tilde{R}$ can be determined through the perimeter of this ring in some medium position in the fill (Fig. 1a). If the ring axis coincides with the average direction (Fig. 1b) of the liquid flow, then the perimeter is equal to $4\pi R$, and if it is perpendicular to it (Fig. 1c), then it is equal to $2H$. The arithmetic mean gives for the effective radius $\tilde{R} = R\left(1 + \frac{H}{2\pi R}\right)$.

Let us replace the volumetric rate of flow in each grain by the total rate of flow through the fill. If the reactor cross-section area is πR_r^2, then for N grains present in the given cross-section $N\tilde{R}^2 = R_\mathrm{r}^2(1-\varepsilon)$ holds. On the other hand, since the water flows only along the grains, $NQ_3 = Q$. Therefore

$$Q_2 = \frac{Q}{4\pi\tilde{R}}\left(\frac{\tilde{R}}{R_\mathrm{r}}\right)^2 \frac{1}{1-\varepsilon} = \mathrm{Re}_\mathrm{r}\frac{\nu}{4}. \tag{5}$$

From relations (4) and (5) we obtain

$$\delta = \left(\frac{3\nu^2}{4\tilde{g}}\mathrm{Re}_\mathrm{r}\right)^{1/3}$$

or

$$\frac{\delta}{\tilde{R}} = \left(\frac{3\nu^2}{4\tilde{g}\tilde{R}^3}\right)^{1/3}\mathrm{Re}_\mathrm{r}^{1/3} = \frac{(3/4)^{1/3}}{\gamma^{1/3}}\mathrm{Re}_\mathrm{r}^{1/3}, \quad \gamma = \left(\frac{\tilde{R}^3\tilde{g}}{\nu^2}\right). \tag{6}$$

Compare the film thickness to the average size of the pores. At $\tilde{R} = 1$ cm $\gamma = 10^7$. Let us take for estimates the typical values of $R_\mathrm{r} = 10$ cm, $\nu = 10^{-2}$ cm²/sec, $\varepsilon = 0.5$, $Q = 20$ cm²/sec. The typical value for Re_r is of the order of 10–20. This means that $\delta/\tilde{R} \approx 0.01$ and the pores at such flow rates are really not completely filled with water and film flow conditions are realized.

Let us estimate the distance x_* at which the film on the ring is stabilized. The thickness of the hydrodynamic boundary layer increases with the coordinate along the flow velocity x according to the law of [6]: $\delta_g = 3^{1/2}2^{3/4}\nu^{1/2} x^{1/4}/\tilde{g}^{1/4}$. At distance x_* it will reach, according to (4), a stable film thickness $\delta = (3Q_2\nu/\tilde{g})^{1/3}$. Setting δ_g equal to δ, we obtain $x_* = \frac{1}{16}\frac{Q_2^{4/3}}{g^{1/3}\nu^{2/3}}$. Compare this value to the grain size:

$$\frac{x_*}{\tilde{R}} = \frac{\mathrm{Re}_r^{4/3}}{4^{10/3}}\left(\frac{\nu^2}{\tilde{g}\tilde{R}^3}\right)^{1/3}\left(\frac{\tilde{R}}{R_r}\right)^{4/3} = 0.01\mathrm{Re}_r^{4/3}\gamma^{-1/3}\left(\frac{\tilde{R}}{R_r}\right)^{4/3} .$$

At $\mathrm{Re}_r = 10$, $\gamma = 10^7$, $\left(\frac{\tilde{R}}{R_r}\right) = 0.1$ we obtain $x_*/\tilde{R} = 10^{-4}$, i.e., the distance at which a stabilized film is formed is very small compared to the grain size. Although the film really begins to develop again on each ring, it suffices to consider precisely the stabilized portion.

Solving the mass transfer problem, we assume that the liquid flowing onto the ring at a very short distance forms a film of thickness δ with a uniformly distributed concentration of the substrate C_b. Since the diffusion boundary layer is much thinner than the water film (from the analysis of dimensions $\delta_D/\delta \approx \mathrm{Sc}^{-1/3}$ a sharper estimate is given below) the diffusion problem can be formulated as follows:

$$\frac{\tilde{g}\delta}{\nu} y \frac{\partial C}{\partial x} = D\frac{\partial^2 C}{\partial y^2}, \quad C = C_b \text{ at } y = 0, \quad C = C_\delta \text{ at } y = \infty . \tag{7}$$

The solution of this problem is of the form [5, 17, 18]

$$\beta = \frac{D\left.\frac{\partial C}{\partial y}\right|_{y=0}}{C_\delta - C_b} = 0.78\frac{D}{\delta}\left(\frac{Q_2}{D}\right)^{1/3}\left(\frac{\delta}{x}\right)^{1/3} . \tag{8}$$

The average value of β for the grain length $2\tilde{R}$ will be

$$\beta = 1.17\frac{D}{\delta}\left(\frac{Q_2}{D}\right)^{1/3}\left(\frac{\delta}{2\tilde{R}}\right)^{1/3} . \tag{9}$$

The δ and Q_2 values should be expressed in terms of the total flow rate (5). Then (9) reduces to

$$\beta = 1.17\frac{D^{2/3}}{\delta^{2/3}}\left(\frac{Q_2}{2\tilde{R}}\right)^{1/3} = 0.62\left(\frac{\tilde{g}^2}{\nu\tilde{R}^3}\right)^{1/9} D^{2/3}\,\mathrm{Re}_r^{1/9} . \tag{10}$$

To represent the law of mass transfer in the criteria form, we rewrite (10) in the form

$$\frac{\beta\tilde{R}}{D} = 0.62\left(\frac{\nu}{D}\right)^{1/3}\left(\frac{\tilde{g}\tilde{R}^3}{\nu^2}\right)^{2/9}\mathrm{Re}_r^{1/9}$$

or

$$\mathrm{Sh} = 0.62\gamma^{2/9}\mathrm{Sc}^{1/3}\,\mathrm{Re}_r^{1/9} . \tag{11}$$

At $\tilde{R} \approx 1$ cm we have $\gamma = 10^7$ and expression (11) takes on the form

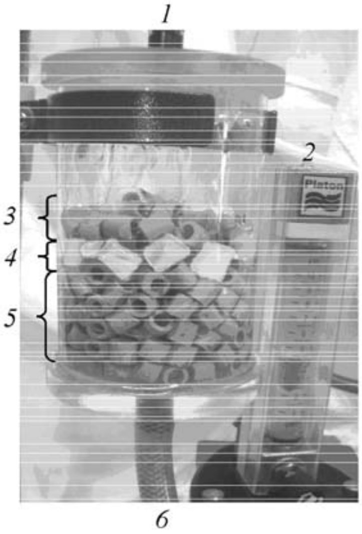


Fig. 2. Facility for measuring the mass transfer coefficient: 1) entrance; 2) flowmeter; 3) protected layer of uncoated rings; 4) coated rings; 5) bed of uncoated rings; 6) drain.

$$\mathrm{Sh} = 22\mathrm{Sc}^{1/3}\,\mathrm{Re}_{\mathrm{r}}^{1/9}\,. \tag{12}$$

Note that at typical values of $\nu = 10^{-2}$ cm^2/sec and $D = 10^{-5}$ cm^2/sec the value of $\mathrm{Sc} = 10^3$.

Defining the diffusion layer thickness as $\delta_D = (C_\delta - C_{\mathrm{b}})\left(\left.\frac{\partial C}{\partial y}\right|_{y=0}\right)^{-1}$, we get $\delta_D/\delta = 0.76\gamma^{1/9}/(\mathrm{Sc}^{1/3}\mathrm{Re}_{\mathrm{r}}^{4/9})$. At the above typical values for γ, Sc, and Re_{r} this gives for δ_D/δ a value of the order of 0.1, which justifies the application here of the "submerged diffusion layer" model.

Experimental Procedure. The mass transfer coefficient in the film flow around Raschig rings in the fill was determined by measuring the loss of the mass of the highly soluble substance applied to the surface of the rings. If dissolving is limited to the solvate removal, then the mass loss of the applied substance is determined by the equation

$$\frac{dm}{dt} = -\beta A\,(C_{\mathrm{s}} - C_{\mathrm{f}})\,. \tag{13}$$

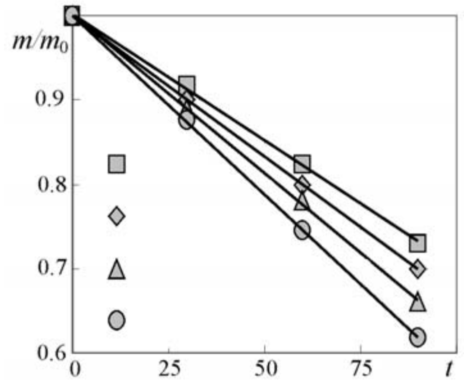
For the substance that is applied to the rings, we chose sugar mixed with honey at a temperature of 80°C in the ratio 2.5:1 to obtain a thick consistency similar to that of modeling clay.

Rings (of height $H = 1.1$ cm, outer diameter $R_{\mathrm{out}} = 0.4$ cm, and inner diameter $R_{\mathrm{in}} = 0.3$ cm) were coated with the given composition manually. Experiments were performed separately for the inner and outer surfaces of the rings. For each experiment, we used ten rings prepared in this way.

Rings coated with the above composition were placed on a fill (of thickness about five times larger than the size of one ring) from uncoated rings in a reactor with a radius of 10 cm and covered on top with a protective layer of rings about twice thicker than the size of one ring (Fig. 2). The orientation of the rings in each sublayer was random. Pure water was supplied from above through a narrow-meshed grid onto the fill in the form of thin jets at a water temperature of 18°C. The flow rate of water was set and controlled by a flowmeter.

We have investigated three variants of ring coating: on the outer surface of the ring, on the inner surface, and on both surfaces.

The sprinkling process was interrupted every 30 sec. Rings covered with honey were removed, placed on dry paper for 3 min, and turned over after 1.5 min. After another 1.5 min they were weighed and placed again for subsequent sprinkling.



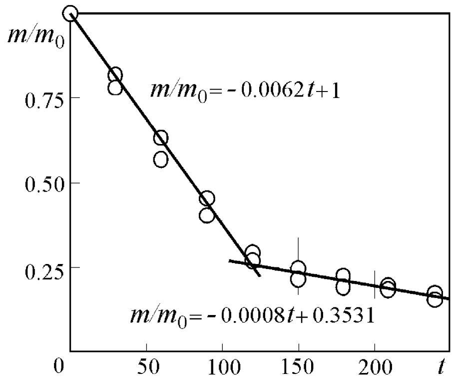


Fig. 3. Time dependence of the coating ablation for various flow rates of water: 1) Q = 7.7; 2) 15.4; 3) 23.11; 4) 38.51 cm^3/sec. t, sec.

Fig. 4. Coating ablation as a function of time. Dots show measurements, straight lines — approximation. t, sec.

For the case of washing rings with a coating on their outer surface, the coating mass decreases directly with time beginning from some initial mass m_0 (Fig. 3). The mass loss rate is the higher, the higher the specific flow rate of water through the reactor. A similar picture is also observed in washing rings with their inner surface coated.

In the case where both surfaces of rings were coated, we can note a moment when the outer surface becomes completely cleaned and further dissolving occurs only on the inner surface (Fig. 4), which is much slower than on the outer surface. The mass loss rate at the final stage of dissolving is one order of magnitude lower than at the initial stage. To separate the effect of joint dissolution on the inner and outer surfaces [15], the basic measurements were made with the solvate applied only to the outer surface.

Additional measurements for the sugar-honey composition have shown that the quantity of water was w = 0.14 of the total weight. The content of sugar in the saturated solution at 20°C is φ = 0.64 of the total weight of the solvate. Thus, $C_s = P_c(1 - w)\varphi$. The C_f value can be taken to be equal to zero because of the rapid change of water by the surface past which it flows. The sugar content in the running-off water is so small that the photometric method failed to detect it.

Data Processing. Equation (13) can be rewritten in the form

$$\beta = \frac{1}{\rho_c A\,(1 - w)}\left|\frac{dm}{dt}\right| . \qquad (14)$$

In the process of water flow around rings, the coating becomes thinner, which has an effect on the washing area which was calculated in the following way.

In the case of coating the outer surface of the rings, the coat thickness L_{out} can be estimated from the balance of the coating substance volume (calculation for one ring):

$$V = \pi H\left[\left(\frac{R_{\text{out}}}{2} + L_{\text{out}}\right) - \left(\frac{R_{\text{out}}}{2}\right)^2\right] = \frac{m}{10\rho_c} .$$

At a sufficiently thin coating ($L_{\text{out}} << R_{\text{out}}$) the expression $L_{\text{out}} = V/(2\pi R_{\text{out}} H)$ can be used. Then the coating surface

$$A_{\text{out}} = 2HR_{\text{out}}\left(1 + \frac{8V}{\pi R_{\text{out}}^2 H}\right). \qquad (15)$$

Likewise, for the case of coating the inner surface of the ring $L_{\text{in}} = 2V/(\pi R_{\text{in}} H)$ and

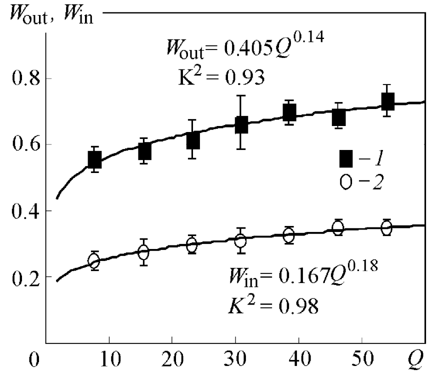


Fig. 5. Portion of the washed surface of rings as a function of the flow rate of the liquid for the outer and inner walls of the ring: 1) outer surface of the ring; 2) inner surface. Q, cm^3/sec.

$$A_{in} = 2\pi H R_{in}\left(1 - \frac{8V}{\pi R_{in}^2 H}\right). \tag{16}$$

True Washed Surface of Rings. It has been noticed visually that at low flow rates of the liquid the ring surface is washed incompletely. Therefore, it is necessary to measure the area of the washed surface of the ring depending on the flow rate. To this end, we applied a mark consisting of 24 equal rectangular sectors to 10 rings.

Before loading the rings into a minireactor, their outer or inner surface was neatly smeared with a thin layer of pure liquid honey to which flour was applied. This was preferred to castor sugar because it is less saturable with water. Upon loading the rings into the reactor water was supplied for 5 sec (to investigate the outer surface) and for 10 sec (to investigate the inner surface). A longer washing time (10 sec) for the inner surface was chosen because the inner surface of the rings is more difficult for access by water.

Upon removal of the rings we calculated: 1) clean (washed) rectangular sectors, 2) partly bared sectors the number of which was divided in two.

It has been established by the measurement data that even at a relatively intensive liquid flow through the reactor space in the film regime (fill pores are not completely filled with water) water does not cover the entire surface of the control group of rings. This effect shows up most vividly in the case of washing the inner surface of the rings, since the flow does not fill the whole of the channel and flow takes place only along its inner bottom surface. The portions of the intensively washed surface $W_{out} = A_{w,out}/A_{out}$ and $W_{in} = A_{w,in}/A_{in}$ depending on the flow rate of the liquid are shown in Fig. 5. Comparative analysis reveals the following features inherent in the process of washing rings by a liquid:

1) the function describing the portion of the washed surface shows a power dependence on the flow rate of water for both the outer wall of the ring and the inner wall, with the washed portion of the outer surface at the same flow rates of the liquid being larger than that of the inner surface;

2) with increasing flow rate of the liquid the portion of the washed surface for the inner walls of the rings increases more rapidly than for the outer surface.

The relative area of the washed surface is probably determined by the Weber number. Under the conditions being considered $\mathrm{We} = Q^2\rho/(\sigma R^2)$. Proceeding from the measurement data, it may be presumed that $A_w/A \propto \mathrm{We}^{0.08}$.

Using (14), it can easily be seen that there is a relation between the "reduced" mass transfer coefficient β determined for the entire surface of the rings A and the "true" β_w taking into account the incomplete washing of the surface A_w:

$$\frac{\beta}{\beta_w} = \frac{A_w}{A} < 1 . \tag{17}$$

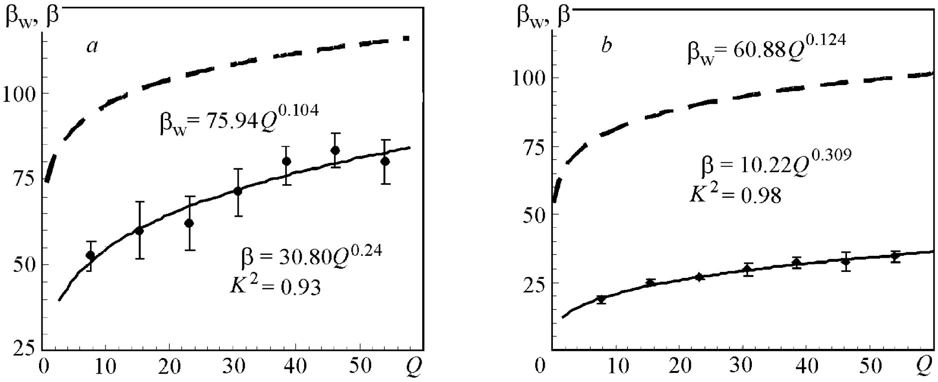


Fig. 6. Mass transfer coefficients to the outer (a) and inner (b) surfaces of rings as a function of the flow rate of the liquid calculated with account (dashed curve) and without account (solid curve) for the incomplete washing of the surface. β_w, β, cm/day; Q, cm^3/sec.

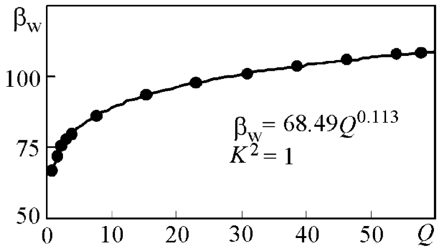


Fig. 7. Mass transfer coefficient in the case of two-sided coating of the ring as a function of the flow rate of the liquid, β_w, cm/day; Q, cm^3/sec.

Mass Transfer Coefficients. Figure 6a shows the dependence of the coefficient of mass transfer to the outer surface of the rings on the flow rate of the liquid. Solid curves present the results without account for the incomplete washing of the surface, and dashed lines — with its account. Analogous results are given in Fig. 6b for the case of coating the inner surface of rings. The powers of the dependences for the mass transfer coefficients to the outer and inner surfaces of rings on the flow rate of the liquid are approximately equal. The parameter values before the powers differ by 40%.

In biotechnology, the mass transfer processes on the outer and inner surfaces of rings proceed simultaneously. The mass transfer coefficient on "average" on the entire surface of the ring can be calculated from the balance of the complete flow from the inner and outer surfaces $\beta(A_{\text{in}} + A_{\text{out}}) = \beta_{\text{in}}A_{\text{in}} + \beta_{\text{out}} + A_{\text{out}}$. As a rule, the difference between A_{in} and A_{out} is small and can be restricted to the arithmetic mean

$$\beta = \frac{\beta_{\text{in}} + \beta_{\text{out}}}{2} . \tag{18}$$

The power approximation $\beta_w(Q)$ shown in Fig. 7 gives $\beta_w = 68.49Q^{0.1134}$. Upon a similar operation the dependence of the portion of the washed surface of the ring on the flow rate of the liquid in the case of both surfaces coated is given in the form

$$\frac{A_w}{A} = 0.2857Q^{0.1567} . \tag{19}$$

Comparison Between Experiment and Theory. The dependence of β_w on the flow rate of the liquid has a power character, with the experimentally obtained power practically coinciding with the theoretical value "1/9" for the mass transfer in a film falling by gravity. The parameters at which the experiment was performed had the values of $R_r = 10$ cm, $\nu = 10^{-2}$ cm^2/sec, $\varepsilon = 0.75$, $\tilde{R} = 0.94$ cm, $D = 0.8$ cm^2/day.

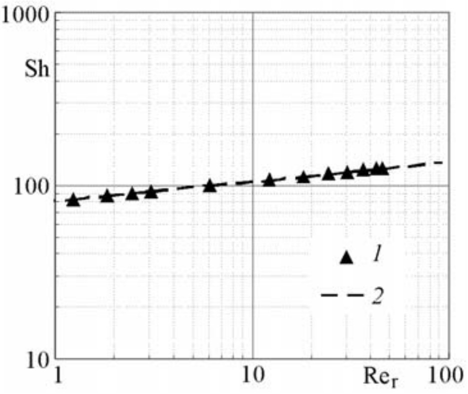


Fig. 8. Comparison between the experimental and theoretical dependences of Sc on Re$_r$: 1) measurements; 2) theory (21).

The constant 0.62 on the right-hand side of equality (10) is conditional since the formula was obtained for a liquid layer running down by a flat surface and, therefore, it should be redetermined by comparing it with the experimental data of [8], which leads to the corrected expression for the mass transfer coefficient

$$\beta_w = 0.43 \left(\frac{\tilde{g}^2}{\nu \tilde{R}^3}\right)^{1/9} D^{2/3} \left(\frac{Q\tilde{R}}{\pi \tilde{R}_r^2 \nu (1-\varepsilon)}\right)^{1/9} \tag{20}$$

or in the criteria form

$$\mathrm{Sh} = 0.43\gamma^{2/9}\,\mathrm{Sc}^{1/3}\,\mathrm{Re}_r^{1/9}\,. \tag{21}$$

CONCLUSIONS

1. According to visual observations, the water flow along grains by gravity at relatively small flow rates in biofilters under the conditions of free air-water surface contact is realized in the film regime.
2. An important fact of the film flow is the incomplete washing of the surface of Raschig rings, and the portion of the washed surface therewith is a function of the flow rate of the liquid.
3. The supply of dissolved substrate to the biofilm in a bioreactor with a fixed granular bed can be modeled by the process of mass transfer in a freely falling film following the law Sh $\propto$ Re$^{1/9}$.

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NOTATION

A, area of the solvate-covered surface, m^2; *C*, substrate (solvate) concentration, kg/m^3; *D*, diffusion coefficient in water, m^2/sec; *g*, gravitational acceleration, m/sec^2; $\tilde{g}$, gravitational acceleration with account for the angle α between the grain surface and the averaged direction of the flow; *H*, height of the Raschig ring, m; *K*, correlation coefficient; *k*, constant; *l*, biofilm thickness, m; *L*, thickness of the Raschig ring surface, coating, m; *m*, solvate mass, kg; *N*, number of grains in a given section of the reactor; *n*, constant; Q_2, flow rate of the liquid per unit width of the film, m^2/sec; Q_3, average volume of the liquid running off the Raschig ring, m^3/sec; *Q*, flow rate of the liquid through the reactor, m^3/sec; *R*, Raschig ring radius, m; R_r, reactor radius, m; $\tilde{R}$, effective size of the Raschig ring, m; Re$_r$, Reynolds number; Sh, Sherwood number; Sc, Schmidt number; *t*, time, sec; *u*, water velocity in the water film, m/sec; *x*, *y*, coordinates along the Raschig ring surface and perpendicular to it, m; x_*, distance at which the film on the ring stabilizes, m; *V*, coating volume, m^3; *w*, water content in the solvate; We, Weber number; *W*, portion of the effectively washed surface; α, angle between the grain surface and the averaged direction of the flow; β, mass transfer coefficient through the water film, m/sec; γ, ratio of the effective radius of the Raschig ring to the viscous-gravitational scale of length in the gravity film; δ, water film thickness, m; δ_D, diffusion layer thickness near the fill grain surface, m; δ_g, thickness of the hydrodynamic boundary layer, m; ε, porosity of the fill in the reactor; θ, constant or some function; ν, kinematic viscosity, m^2/sec; ρ, water density, kg/m^3; ρ_c, coating density, kg/m^3; σ, surface tension, N/m; φ, sugar content in saturated sugar solution. Subscripts: b, solvate–liquid interface; c, coating; D, diffusion; f, far from the washed surface; g, hydrodynamic; in, on the inner surface; out, on the outer surface; s, saturation; r, reactor; w, on the effectively washed surface; 0, initial; δ, on the water film surface.

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